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Codes Correcting Blockwise *s*-Periodic Errors

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ABSTRACT

In this paper, we obtain lower and upper bounds for linear codes which are capable of correcting the errors blockwise that occur during the process of transmission. The kinds of errors considered are known as *s*-periodic errors. Illustrations for such kind of codes have also been provided.

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1. INTRODUCTION

In most memory and storage system, the information is stored in various parts of the code length, known as sub-blocks. So, when error occurs in such a system, it does in a few places of the same sub-block and the pattern of errors is known. Thus, when we consider error correction, we correct errors which occur within the same sub-block. Hence there is a need to study block-wise error correcting (BEC) codes. In fact, block-wise correction of error is an extension of the concept- Error Location, introduced by Wolf and Elspas [6]. They obtained results in the form of bounds over the number of parity-check digits required for binary codes capable of detecting and locating a single sub-block containing random errors. A study of such error locating codes in which errors occur in the form of bursts was made by Dass [1]. Further, these results were extended to the codes correcting burst errors occurring within a sub-block (refer Dass and Tyagi [4]). Similar studies were also done by Dass and Madan [3], Dass and Arora [2]. The development of codes correcting errors within a sub-block improves the efficiency of the communication channel.

In coding theory, many types of error patterns have been dealt with and codes have been constructed to combat such error patterns. Though the errors are generally classified mainly in two categories - random errors and burst errors, it has also been observed that the occurrence of errors may follow a pattern, different from these two. The pattern is such that error repeats after certain fixed interval. In certain communication channel like Astrophotography [11] where small mechanical error occurs periodically in the accuracy of the tracking in a motorized mount that results small movements of the target that can spoil long-exposure images, even if the mount is perfectly polar-aligned and appears to be tracking perfectly in short tests. It repeats at a regular interval - the interval being the amount of time it takes the mount's drive gear to complete one revolution. This type of errors is termed as *periodic* error. It was in this spirit that the codes detecting and correcting such errors were developed by Tyagi and Das ([8], [9]). An *s*-periodic error may be defined as follows:

Definition: *An s-periodic error is an n-tuple whose non zero components are located at a gap of s positions and the number of its starting positions is among the first s+1 components, where s = 1, 2, 3,...,(n-1).*

For *s* =1, the 1-*periodic* errors are the vectors where error may occur in 1st, 3rd, 5th,...... positions or 2nd, 4th, 6th,...... positions. For example, in a vector of length 8, 1-*periodic* error vectors are of the type 10101000, 00101000, 0010101, 10101010, 10001010, 01010101, 01000101, 00000101, 00000001 etc.

For *s* =2, the 2-*periodic* error vectors are those where error may occur in 1st, 4th, 7th,...... positions or 2nd, 5th, 8th,......positions or 3rd, 6th, 9th,.......positions. The 2-*periodic* error vectors may look like 10010010, 10000010, 00010010, 01001001, 01000001, 01000000, 00001001, etc in a vector of length 8.

For *s* =3, in a code length 8, the 3-*periodic* error vectors are 10001000, 01000100, 00100010, 00010001, 10000000, 01000000 etc.

In a paper [7], Das obtains bounds on codes over *GF(q)* that corrects different type of *periodic* errors in two sub-blocks. This paper extends the work and obtains bounds for codes correcting periodic errors in any finite number (say *b*) of sub-blocks.

The rest of the paper is organized as follows.
Section 2 gives a lower bound on the number of check digits required for the existence of a linear code over GF(q) capable of correcting errors that are in the form of periodic errors within a sub-block. In section 3, we derive an upper bound on the number of check digits which ensures the existence of such a code. Illustrations of such codes over GF(2) have also been given.

In the following, we shall consider a linear code to be a subspace of *n*-tuples over GF(q). The block of *n* digits, consisting of *r* check digits and *k* = *n* - *r* information digits, is considered to be divided into *b* mutually exclusive sub-blocks. Each sub-block contains *t*=*n/b* digits.

We note that an (*n*, *k*) linear code capable of correcting an error requires the syndromes of any two vectors to be different irrespective of whether they belong to the same sub-block or different sub-blocks. So, in order to correct *s*-periodic errors lying within a sub-block the following conditions need to be satisfied:

(a) The syndrome resulting from the occurrence of an *s*-periodic error must be distinct from the syndrome resulting from any other *s*-periodic errors within the *same* sub-block.
(b) The syndrome resulting from the occurrence of *s*-periodic errors within a single sub block must be distinct from the syndrome resulting likewise from any *s*-periodic errors within *any other* sub block.

2. LOWER BOUND

In the following, we derive a lower bound on the number of check digits required for the existence of a linear code over GF(q) capable of correcting errors that are *s*-periodic errors within a sub-block. The proof follows the technique used in the theorem 4.16, Peterson and Weldon [10].

Theorem 1 *The number of parity check digits r in an (n, k) linear code subdivided into b sub-blocks of length t each, that corrects s-periodic error lying within a single sub-block is at least*

$$loq_q\left\{1+b\sum_{i=0}^{s}(q^{k_i}-1)\right\} \text{ where } k_i=\left\lceil\frac{t-i}{s+1}\right\rceil. \quad (1)$$

Proof. Let there be an (*n*, *k*) linear code vector over GF(q) that corrects all *s*-periodic errors within a single corrupted sub-block. The maximum number of distinct syndromes available using *r* check bits is q^r. The proof proceeds by first counting the number of syndromes that are required to be distinct by condition (a) and (b) and then setting this number less than or equal to q^r. First we consider a sub-block, say *i*-th sub-block of length *t*. Since the code is capable of correcting all errors which are *s*-periodic errors within a single sub-block, any syndrome produced by an *s*-periodic error in a given sub-block must be different from any such syndrome likewise resulting from *s*-periodic errors in the same sub-block by condition (a).

Also by condition (b), syndromes produced by *s*-periodic errors in different sub-blocks must be distinct. Thus the syndromes produced by *s*-periodic errors, whether in the same sub-block or in different sub-blocks should be distinct.
Since there are *b* sub-blocks and number of *s*-periodic errors in a vector of length *t* (Tyagi and Das [9]) is

$$\sum_{i=0}^{s}(q^{k_i}-1) \text{ where } k_i=\left\lceil\frac{t-i}{s+1}\right\rceil. \tag{2}$$

So we must have at least $1+b\sum_{i=0}^{s}(q^{k_i}-1)$ distinct syndromes, including the all zero syndrome.
Therefore we must have

$$q^r \geq 1+b\sum_{i=0}^{s}(q^{k_i}-1) \tag{3}$$

Or,

$$r \geq loq_q\left\{1+b\sum_{i=0}^{s}(q^{k_i}-1)\right\}$$

$$\text{where } k_i=\left\lceil\frac{t-i}{s+1}\right\rceil.$$

Remark 1 For $b = 1$, the bound reduces to

$$r \geq loq_q\left\{1+\sum_{i=0}^{s}(q^{k_i}-1)\right\} \text{ where } k_i=\left\lceil\frac{n-i}{s+1}\right\rceil.$$

which coincides with the necessary condition for the existence of a code correcting all *s*-periodic errors (refer Theorem 1, Tyagi and Das [9]).

3. UPPER BOUND

In the following result, we derive an upper bound on the number of check digits required for the existence of such a code. The proof is based on the technique used to establish Varshamov-Gilbert-Sacks bound by constructing a parity check matrix for such a code (refer Sacks [5], also Theorem 4.7, Peterson and Weldon [10]). This technique not only ensures the existence of such a code but also gives a method for construction of the code.

Theorem 2 *An (n, k) linear code over GF(q) capable of correcting an s-periodic error, occurring within a single sub-block of length t can always be constructed using r check digits where r is the smallest integer satisfying the inequality*

$$q^r > q^p\left\{\sum_{i=1}^{s}q^{k_i}-(s-1)+(b-1)\sum_{i=0}^{s}(q^{k_i}-1)\right\} \tag{4}$$

$$\text{where } p=\left\lceil\frac{t}{s+1}-1\right\rceil \text{ and } k_i=\left\lceil\frac{t-i}{s+1}\right\rceil.$$

Proof. In order to prove the existence of such a code we construct an $(n-k)\times n$ parity check matrix *H* for such a code by a synthesis procedure as follows:
After adding $(b-1)t$ columns appropriately corresponding to the first $(b-1)$ sub-blocks, suppose that we have added the first j-1 columns h_1, h_2, . . ., h_{j-1} (where $j > s+1$) of the *b*-th sub-block. We now lay down the condition to add the *j*-th column h_j as follows:

According to the condition (a), for the correction of *s*-periodic errors within a single sub-block, the syndrome of any *s*-periodic errors within any sub-block must be different from the syndrome resulting from any other *s*-periodic errors within the same sub-block.

So h_j can be added provided it is not a linear combination of previous s-periodic columns $h_{j-(s+1)}$, $h_{j-2(s+1)}$, . . . , $h_{j-p(s+1)}$ where $p = \left\lceil \frac{j}{s+1} - 1 \right\rceil$, together with any other linear combination of s-periodic columns among the j-1 columns of the b-th sub-block. i.e.,

$$h_j \neq \sum_{i=1}^{p} u_i h_{j-i(s+1)} + \sum_{i=0}^{a_m - 1} v_i h_{j-m-i(s+1)}; \quad m = 1, 2, \ldots, s. \tag{5}$$

where $u_i, v_i \in GF(q)$, $p = \left\lceil \frac{j}{s+1} - 1 \right\rceil$ and $a_m = \left\lceil \frac{j-m}{s+1} \right\rceil$.

The number of possible linear combinations of R.H.S. of (5) (refer Tyagi and Das [9]), including the zero vector, is

$$q^p \sum_{i=1}^{s} q^{a_i} - (s-1)q^p . \tag{6}$$

Now according to condition (b), the syndrome of any s-periodic error within a sub-block must be different from the syndrome resulting from any s-periodic error within any *other* sub-block. In view of this, h_j ($j>s+1$) can be added provided that

$$h_j \neq \sum_{i=1}^{p} u_i h_{j-i(s+1)} + \sum_{i=0}^{k_m - 1} v_i h_{((l-1)t+m)+i(s+1)}; \quad m = 1, 2, \ldots, s. \tag{7}$$

where $u_i, v_i \in GF(q)$, $p = \left\lceil \frac{j}{s+1} - 1 \right\rceil$, $k_m = \left\lceil \frac{t-m}{s+1} \right\rceil$ and $h_{(l-1)t+m}$'s are the columns corresponding to l-th sub-block among the b-1 sub-blocks.

The number of ways in which the coefficients u_i's can be selected is q^p and to enumerate the coefficients v_i's is equivalent to enumerate the number of s-periodic errors in a vector of length t.
This number of s-periodic errors within a sub-block of length t, excluding the zero-vector, is given by (2).
Since there are b-1 previously chosen sub-blocks, therefore number of such linear combinations becomes

$$(b\text{-}1) \times \text{expr.}(2).$$

So, the number of linear combinations to which h_j can not be equal to is the product

$$q^p.(b\text{-}1) \times \text{expr.}(2). \tag{8}$$

Thus for the block-wise correction of s-periodic errors, the number of linear combinations to which h_j can not be equal to is the sum of linear combinations computed in (6) and (8).
At worst all these combinations might yield distinct sum.
Therefore h_j can be added to the b-th sub-block of H provided that

$$q^r > \text{expr. (6)+ expr. (8)}.$$

or,

$$q^r > q^p \sum_{i=1}^{s} q^{a_i} - (s-1)q^p + q^p (b-1) \sum_{i=0}^{s} (q^{k_i} - 1)$$

or,

$$q^r > q^p \left\{ \sum_{i=1}^{s} q^{a_i} - (s-1) + (b-1) \sum_{i=0}^{s} (q^{k_i} - 1) \right\} \tag{9}$$

where $p = \left\lceil \frac{j}{s+1} - 1 \right\rceil$, $a_i = \left\lceil \frac{j-i}{s+1} \right\rceil$ and $k_i = \left\lceil \frac{t-i}{s+1} \right\rceil$.

Replacing j by t for the completion of b th sub-block (t being the length of the sub-block) gives the inequality (4) stated in the theorem.

Remark 2 For b = 1, the bound reduces to

$$q^r > q^p \left\{ \sum_{i=1}^{s} q^{k_i} - (s-1) \right\} \text{ where } p = \left\lceil \frac{t}{s+1} - 1 \right\rceil \text{ and } k_i = \left\lceil \frac{t-i}{s+1} \right\rceil$$

which coincides with the bound (refer Theorem 2, Tyagi and Das [9]).

4. ILLUSTRATIONS

Example 1. By taking s=1, t=3, b=3, q=2. The necessary condition - inequality (3) of theorem 1, gives rise to a (9, 5) linear code over GF(2) we construct the following 4×9 parity check matrix H.

$$H = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

The null space of this matrix can be used as a code to correct 1-periodic error within a sub-block of length t = 3. It may be easily verified from error pattern- syndromes table that:

(i) Syndromes of all 1- periodic errors within any one sub-block are all non-zero.

(ii) The syndrome of a 1-periodic error within any sub-block is different from the syndrome of a 1- periodic error within the *same* sub-block.

(iii) The syndrome of a 1-periodic error within any sub-block is different from the syndrome of a 1- periodic error within *any other* sub-block.

Example 2. For a (14, 7) linear code over GF(2), we construct the following (7, 14) parity check matrix H, according to the synthesis procedure given in the proof of Theorem 2 by taking s=2, t=7, b=2.

$$H = \begin{bmatrix} 1&1&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&1&1&0&0&0&1&0&0&1&0&0 \\ 0&0&1&0&0&0&1&0&0&1&0&1&0&0 \\ 0&1&0&0&0&1&0&1&1&1&0&0&1&0 \\ 0&0&0&0&1&0&0&0&0&0&1&1&1&0 \\ 0&0&1&0&0&0&0&0&0&1&0&0&0&1 \\ 0&0&0&0&0&1&0&0&0&0&0&0&1&0 \end{bmatrix}$$

The null space of this matrix can be used as a code to correct 2-periodic error within a sub-block of length t=7. It may be easily verified from error pattern-syndromes table that the syndromes of 2-periodic errors are nonzero, distinct within the same sub block or any other sub block.

For illustration we take the **Example 2** and verify from the error pattern-syndrome **table 1** that all the 3 cases are true.

Table 1: Error patterns and corresponding syndromes blockwise

Error patterns	Syndromes	Error patterns	Syndromes
1st sub-block		2nd sub-block	
1000000 0000000	1000000	0000000 1000000	0001000
0001000 0000000	0100000	0000000 0001000	0000100
0000001 0000000	0010000	0000000 0000001	0000010
1001000 0000000	1100000	0000000 1001000	0001100
1000001 0000000	1010000	0000000 1000001	0001010
1001001 0000000	1110000	0000000 1001001	0001110
0001001 0000000	0110000	0000000 0001001	0000110
0100000 0000000	1001000	0000000 0100000	1101000
0000100 0000000	0100100	0000000 0000100	0110100
0100100 0000000	1101100	0000000 0100100	1011100
0010000 0000000	0010010	0000000 0010000	0011010
0000010 0000000	0001001	0000000 0000010	0001101
0010010 0000000	0011011	0000000 0010010	0010111

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